Hoofdstuk 8: Complexe getallen.

8.1 Rekenen met complexe getallen.

<u>Opgave 1:</u>

a. I: $2x+5=13$

$2x=8$

$x=4$

II: $2x+5=3$

$2x=-2$

$x=-1$

III: $2x+5=8$

$2x=3$

$x=1\tfrac{1}{2}$

IV: $x^2+5=8$

$x^2=3$

$x=\sqrt{3} \quad \vee \quad x=-\sqrt{3}$

b. III en IV

<u>Opgave 2:</u>

$(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})=3-2=1$ dus zit in N

$(-2)^3=-8$ dus zit in Z

$1\tfrac{1}{2}$ zit in Q

$\sqrt{6\tfrac{1}{4}}=2\tfrac{1}{2}$ dus zit in Q

π^2 zit in R

<u>Opgave 3:</u>

a. $x^2+x=6$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3 \quad \vee \quad x=2$

b. $x^2+x=4$

$x^2+x-4=0$

$x=\dfrac{-1\pm\sqrt{17}}{2}$ dus geen oplossingen in Q

c. $x^3-x=0$

$x(x^2-1)=0$

$x=0 \quad \vee \quad x^2=1$

$x=0 \quad \vee \quad x=-1 \quad \vee \quad x=1$

d. $x^3-2x=0$

$x(x^2-2)=0$

$x=0 \quad \vee \quad x^2=2$

dus $x=0$

e. $x^4 - 9x = 0$

$x(x^3 - 9) = 0$

$x = 0 \quad \vee \quad x^3 = 9$

dus $x = 0$

f. $x^4 - 9x^2 = 0$

$x^2(x^2 - 9) = 0$

$x^2 = 0 \quad \vee \quad x^2 = 9$

$x = 0 \quad \vee \quad x = 3 \quad \vee \quad x = -3$

<u>Opgave 4:</u>

a. $(x+3)^2 = 7$

$x + 3 = \sqrt{7} \quad \vee \quad x + 3 = -\sqrt{7}$

$x = -3 + \sqrt{7} \quad \vee \quad x = -3 - \sqrt{7}$

b. $(x+3)^2 = -7$

stel $p = x + 3$ dan $p^2 = -7$, deze vergelijking heeft geen oplossingen

<u>Opgave 5:</u>

a. $3x + 5i + 3 = 2i - x$

$4x = -3 - 3i$

$x = -\frac{3}{4} - \frac{3}{4}i$

b. $2x^2 + 10 = 0$

$2x^2 = -10$

$x^2 = -5$

$x = i\sqrt{5} \quad \vee \quad x = -i\sqrt{5}$

c. $(x+2)^2 + 10 = 0$

$(x+2)^2 = -10$

$x + 2 = i\sqrt{10} \quad \vee \quad x + 2 = -i\sqrt{10}$

$x = -2 + i\sqrt{10} \quad \vee \quad x = -2 - i\sqrt{10}$

d. $x^2 - 10x + 40 = 0$

$(x-5)^2 - 25 + 40 = 0$

$(x-5)^2 = -15$

$x - 5 = i\sqrt{15} \quad \vee \quad x - 5 = -i\sqrt{15}$

$x = 5 + i\sqrt{15} \quad \vee \quad x = 5 - i\sqrt{15}$

e. $x^2 + 8x + 14 = 0$

$(x+4)^2 - 16 + 14 = 0$

$(x+4)^2 = 2$

$x + 4 = \sqrt{2} \quad \vee \quad x + 4 = -\sqrt{2}$

$x = -4 + \sqrt{2} \quad \vee \quad x = -4 - \sqrt{2}$

f. $(x+3)^2 = -16$

$x + 3 = 4i \quad \vee \quad x + 3 = -4i$

$x = -3 + 4i \quad \vee \quad x = -3 - 4i$

Opgave 6:

a. $(x-3)^2+x=0$

$x^2-6x+9+x=0$

$x^2-5x+9=0$

$(x-2\tfrac{1}{2})^2-6\tfrac{1}{4}+9=0$

$(x-2\tfrac{1}{2})^2=2\tfrac{3}{4}$

$x-2\tfrac{1}{2}=\tfrac{1}{2}i\sqrt{11} \quad \vee \quad x-2\tfrac{1}{2}=-\tfrac{1}{2}i\sqrt{11}$

$x=2\tfrac{1}{2}+\tfrac{1}{2}i\sqrt{11} \quad \vee \quad x=2\tfrac{1}{2}-\tfrac{1}{2}i\sqrt{11}$

b. $(2x+3)^2+10=0$

$(2x+3)^2=-10$

$2x+3=i\sqrt{10} \quad \vee \quad 2x+3=-i\sqrt{10}$

$2x=-3+i\sqrt{10} \quad \vee \quad 2x=-3-i\sqrt{10}$

$x=-1\tfrac{1}{2}+\tfrac{1}{2}i\sqrt{10} \quad \vee \quad x=-1\tfrac{1}{2}-\tfrac{1}{2}i\sqrt{10}$

c. $\tfrac{1}{3}x+10+2i=\tfrac{1}{4}x+12-5i$

$\tfrac{1}{12}x=2-7i$

$x=24-84i$

d. $4x^2+4x+7=0$

$$x=\frac{-4\pm\sqrt{-96}}{8}=\frac{-4\pm 4i\sqrt{6}}{8}$$

$x=-\tfrac{1}{2}+\tfrac{1}{2}i\sqrt{6} \quad \vee \quad x=-\tfrac{1}{2}-\tfrac{1}{2}i\sqrt{6}$

Opgave 7:

a. $(2+i)(10-5i)=20-10i+10i-5i^2=20+5=25$

b. $(a+bi)(c+di)=ac+adi+bci+bdi^2=ac-bd+(ad+bc)i$

Opgave 8:

a. $2+5i+4-6i=6-i$

b. $(5-i)(5+i)=25-i^2=25+1=26$

c. $(2+i)^2=4+4i+i^2=4+4i-1=3+4i$

d. $i(6+7i)=6i+7i^2=-7+6i$

e. $i^5=i\cdot i^2\cdot i^2=i\cdot -1\cdot -1=i$

f. $(1+i)(6-i)+(3-i)(3+2i)=6+5i-i^2+9+3i-2i^2=$

$6+5i+1+9+3i+2=$

$18+8i$

Opgave 9:

a. $$\frac{2}{1+i}=\frac{2}{1+i}\cdot\frac{1-i}{1-i}=\frac{2-2i}{1-i^2}=\frac{2-2i}{2}=1-i$$

b. $$\frac{2+3i}{i}=\frac{2+3i}{i}\cdot\frac{i}{i}=\frac{2i+3i^2}{i^2}=\frac{2i-3}{-1}=3-2i$$

c. $\frac{2+i}{2-i}=\frac{2+i}{2-i}\cdot\frac{2+i}{2+i}=\frac{4+4i+i^2}{4-i^2}=\frac{4+4i-1}{4+1}=\frac{3+4i}{5}=\frac{3}{5}+\frac{4}{5}i$

d. $\frac{3+5i}{12+5i}=\frac{3+5i}{12+5i}\cdot\frac{12-5i}{12-5i}=\frac{36+45i-25i^2}{144-25i^2}=\frac{36+45i+25}{144+25}=\frac{61}{169}+\frac{45}{169}i$

e. $\frac{2i}{1+3i}=\frac{2i}{1+3i}\cdot\frac{1-3i}{1-3i}=\frac{2i-6i^2}{1-9i^2}=\frac{2i+6}{1+9}=\frac{6}{10}+\frac{2}{10}i=\frac{3}{5}+\frac{1}{5}i$

f. $(2+3i)\cdot\frac{3}{2+i}=\frac{6+9i}{2+i}=\frac{6+9i}{2+i}\cdot\frac{2-i}{2-i}=\frac{12+12i-9i^2}{4-i^2}=\frac{12+12i+9}{4+1}=4\frac{1}{5}+2\frac{2}{5}i$

Opgave 10:

a. $(3+4i)^2=9+24i+16i^2=9+24i-16=-7+24i$

b. $\frac{3+i}{3+4i}=\frac{3+i}{3+4i}\cdot\frac{3-4i}{3-4i}=\frac{9-9i-4i^2}{9-16i^2}=\frac{9-9i+4}{9+16}=\frac{13}{25}-\frac{9}{25}i$

c. $(2+i)^2-(2-i)^2=4+4i+i^2-(4-4i+i^2)=4+4i-1-4+4i+1=8i$

d. $i^2(i^3-i^4-i^5)=-1(i^3-i^4-i^5)=-i^3+i^4+i^5=-i\cdot i^2+i^2\cdot i^2+i^2\cdot i^2\cdot i=$
$=-i\cdot-1+-1\cdot-1+-1\cdot-1\cdot i=i+1+i=1+2i$

e. $\frac{5}{2+3i}=\frac{5}{2+3i}\cdot\frac{2-3i}{2-3i}=\frac{10-15i}{4-9i^2}=\frac{10-15i}{13}=\frac{10}{13}-\frac{15}{13}i$

f. $i+i^2+i^3+...+i^{10}=i-1-i+1+i-1-i+1+i-=-1+i$
want $i^2=-1$, $i^3=i^2\cdot i=-i$, $i^4=i^2\cdot i^2=-1\cdot-1=1$, $i^5=i\cdot i^4=i$ etc.

Opgave 11:

$\frac{a+bi}{c+di}=\frac{a+bi}{c+di}\cdot\frac{c-di}{c-di}=\frac{ac-adi+bci-bdi^2}{c^2-d^2i^2}=\frac{ac+bd+bci-adi}{c^2+d^2}=\frac{ac+bd-(ad-bc)i}{c^2+d^2}$

Opgave 12:

a. $(a+bi)(a-bi)=a^2-b^2i^2=a^2+b^2$

b. $(3+4i)(3-4i)=9-16i^2=9+16=25$
$(100-200i)(100+200i)=10000-40000i^2=10000+40000=50000$
$(0{,}4i+0{,}3)(0{,}3-0{,}4i)=-0{,}16i^2+0{,}09=0{,}16+0{,}09=0{,}25$

Opgave 13:

a. $z=a+bi$ $\quad\overline{z=a-bi}$
$\frac{z+\bar{z}}{2}=\frac{a+bi+a-bi}{2}=\frac{2a}{2}=a=\mathrm{Re}(z)$

b. $\frac{z-\bar{z}}{2i}=\frac{a+bi-(a-bi)}{2i}=\frac{a+bi-a+bi}{2i}=\frac{2bi}{2i}=b=\mathrm{Im}(z)$

c. $\bar{\bar{z}}=\overline{(\bar{z})}=\overline{(a-bi)}=a+bi=z$

Opgave 14:

a. $\overline{z_1+z_2}=\overline{a+bi+c+di}=\overline{(a+c)+(b+d)i}=a+c-(b+d)i=a+c-bi-di=$
$a-ci+b-di=\overline{z_1}+\overline{z_2}$

b. $z_1 \cdot z_2 = (a+bi)(c+di) = ac - bd + (ad+bc)i$

$\overline{z_1 \cdot z_2} = ac - bd - (ad+bc)i = ac - bd - adi - bci$

$\overline{z_1} \cdot \overline{z_2} = (a-bi)(c-di) = ac - bd - adi - bci = \overline{z_1 \cdot z_2}$

c. $\dfrac{z_1}{z_2} = \dfrac{a+bi}{c+di} = \dfrac{ac+bd-(ad-bc)i}{c^2+d^2}$

$\overline{\left(\dfrac{z_1}{z_2}\right)} = \dfrac{ac+bd+(ad-bc)i}{c^2+d^2}$

$\dfrac{\overline{z_1}}{\overline{z_2}} = \dfrac{a-bi}{c-di} = \dfrac{a-bi}{c-di} \cdot \dfrac{c+di}{c+di} = \dfrac{ac+bd+adi-bci}{c^2+d^2} = \dfrac{ac+bd+(ad-bc)i}{c^2+d^2} = \overline{\left(\dfrac{z_1}{z_2}\right)}$

Opgave 15:

a. $\overline{(4+3i)^2} = \overline{16+24i-9} = \overline{7+24i} = 7-24i$

b. $\dfrac{4+3i}{2+i} = \dfrac{4+3i}{2+i} \cdot \dfrac{2-i}{2-i} = \dfrac{8+2i-3i^2}{4-i^2} = \dfrac{11+2i}{5} = \frac{11}{5} + \frac{2}{5}i$

$\overline{\left(\dfrac{4+3i}{2+1}\right)} = \overline{\frac{11}{5} + \frac{2}{5}i} = \frac{11}{5} - \frac{2}{5}i$

c. $\dfrac{\overline{3+4i}}{3-4i} = \dfrac{3-4i}{3-4i} = 1$

8.2 Het complexe vlak

Opgave 16:

a. $|(2,1)| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$|(1,3)| = \sqrt{1^2 + 3^2} = \sqrt{10}$

b. $\tan\alpha = \frac{1}{2}$ dus $\alpha = 27°$

$\tan\beta = \frac{3}{1}$ dus $\beta = 72°$

c. nee, $|(3,4)| = \sqrt{3^2 + 4^2} = 5$

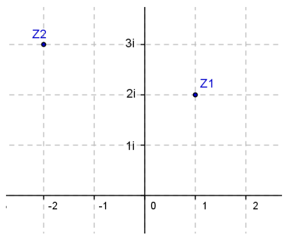
$|(2,1)| + |(1,3)| = \sqrt{5} + \sqrt{10}$

d. $\tan\gamma = \frac{4}{3}$ dus $\gamma = 53°$

nee, $\dfrac{\alpha + \beta}{2} = 49{,}1°$

Opgave 17:

a.

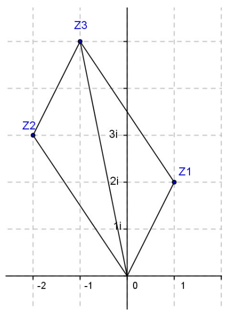


$|z_1| = \sqrt{1^2 + 2^2} = \sqrt{5}$

$|z_2| = \sqrt{(-2)^2 + 3^2} = \sqrt{13}$

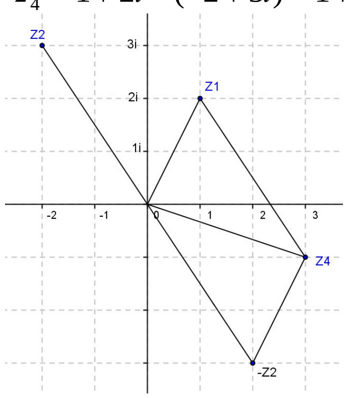
b. $z_3 = 1 + 2i + -2 + 3i = -1 + 5i$

c.



d. $z_4 = 1 + 2i - (-2 + 3i) = 1 + 2i + 2 - 3i = 3 - i$

e.

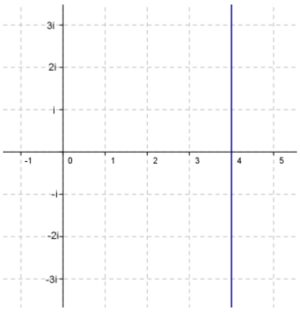


f. $z_5 = (1 + 2i)(-2 + 3i) = -2 + 3i - 4i - 6 = -8 - i$

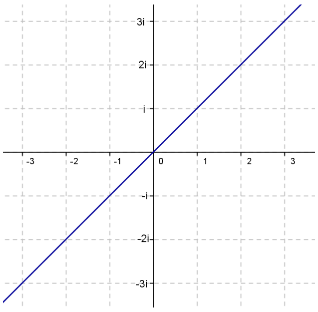
g. $|z_1 \cdot z_2| = \sqrt{(-8)^2 + (-1)^2} = \sqrt{65} = \sqrt{5} \cdot \sqrt{13} = |z_1| \cdot |z_2|$

Opgave 18:

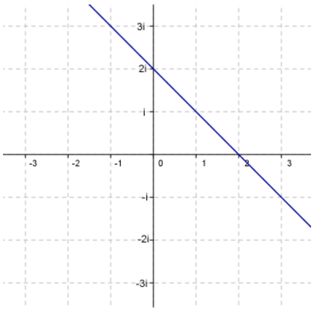
a.



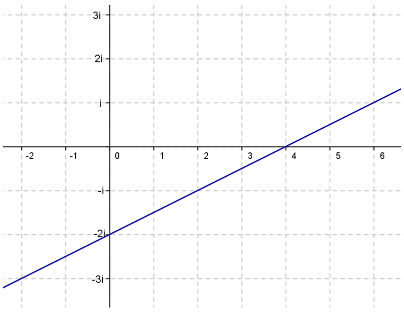
b.



c.

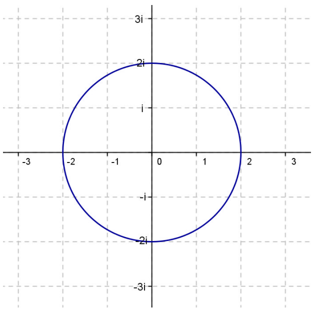


d.

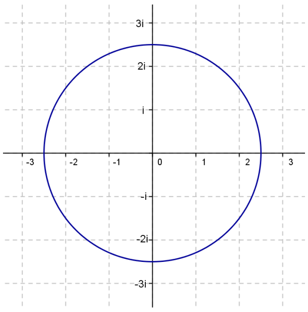


Opgave 19:

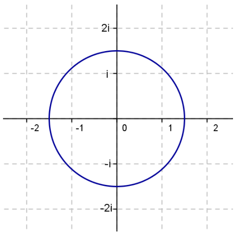
a.



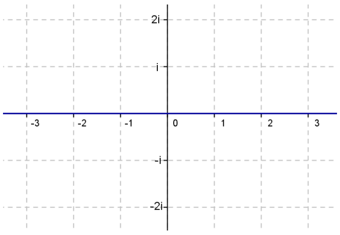
b.



c. $z \cdot \bar{z} = (a+bi)(a-bi) = a^2 + b^2 = 2\frac{1}{4}$



d. $z = \bar{z}$ dus $b = 0$ dus de horizontale as



Opgave 20:

$z \cdot \bar{z} = (a+bi)(a-bi) = a^2 + b^2 = (\sqrt{a^2+b^2})^2 = |z|^2$

Opgave 21:

$z_1 \cdot z_2 = (a+bi)(c+di) = ac + adi + bci - bd = ac - bd + (ad+bc)i$

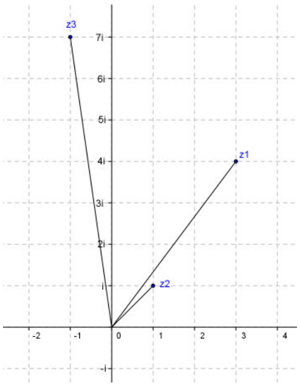
$$
\begin{aligned}
|z_1 \cdot z_2| &= \sqrt{(ac-bd)^2 + (ad+bc)^2} \\
&= \sqrt{a^2c^2 - 2abcd + b^2d^2 + a^2d^2 + 2abcd + b^2c^2} \\
&= \sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2}
\end{aligned}
$$

$$
\begin{aligned}
|z_1| \cdot |z_2| &= \sqrt{a^2+b^2}\sqrt{c^2+d^2} \\
&= \sqrt{(a^2+b^2)(c^2+d^2)} \\
&= \sqrt{a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2}
\end{aligned}
$$

Opgave 22:

a. $z_3 = (3+4i)(1+i) = 3+3i+4i-4 = -1+7i$

b.



c. $\tan z_1 = \frac{4}{3}$ dus $\angle z_1 = 53{,}1°$

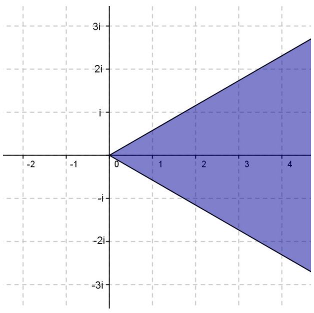
$\tan z_2 = \frac{1}{1}$ dus $\angle z_2 = 45°$

$\tan z_3 = \frac{7}{-1} = -7$ dus $\angle z_3 = 98{,}1°$

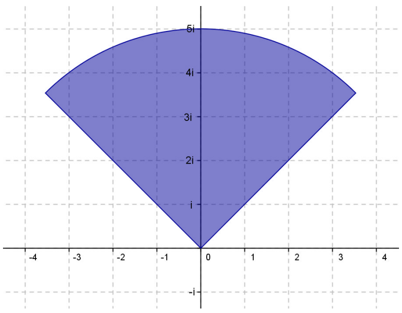
$\angle z_1 + \angle z_2 = \angle z_3$

Opgave 23:

a.



b.



Opgave 24:

a. $|z| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$\tan \varphi = \frac{2}{2} = 1$

$\varphi = 45°$

b. $z = (1-i)^6 = 8i$

$|z| = 8$

$\varphi = 90°$

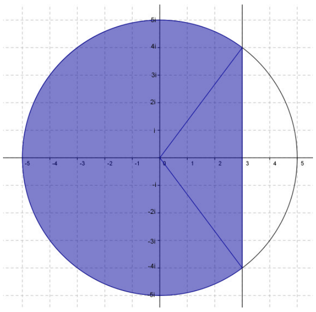
c. $|z| = \sqrt{\cos^2 40° + \sin^2 40°} = \sqrt{1} = 1$

$$\tan\varphi = \frac{\sin 40°}{\cos 40°} = \tan 40°$$
$$\varphi = 40°$$

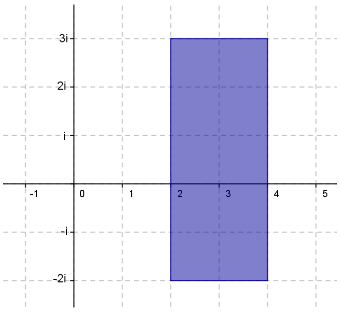
d. $$|z| = \sqrt{(5\cos 140°)^2 + (5\sin 140°)^2}$$
$$= \sqrt{25\cos^2 140° + 25\sin^2 140°}$$
$$= \sqrt{25(\cos^2 140° + \sin^2 140°)}$$
$$= \sqrt{25} = 5$$
$$\tan\varphi = \frac{5\sin 140°}{5\cos 140°} = \tan 140°$$
$$\varphi = 140°$$

Opgave 25:

a.

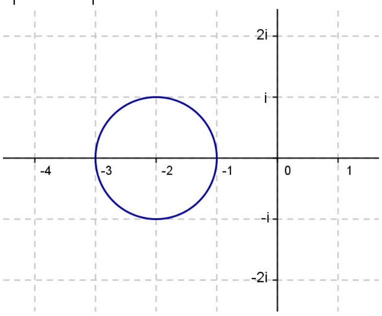


b.

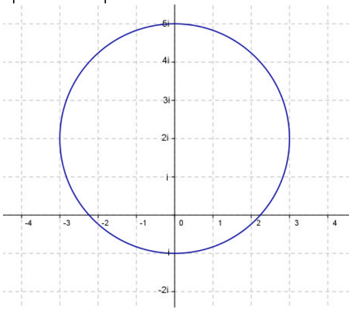


Opgave 26:

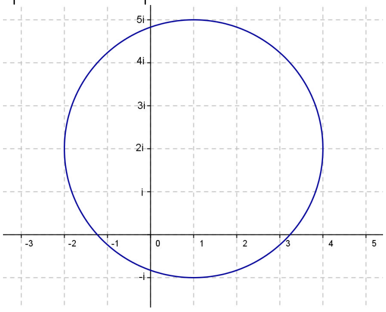
a. $|z+2| = 1$ is een cirkel met middelpunt $(-2,0)$ en straal 1



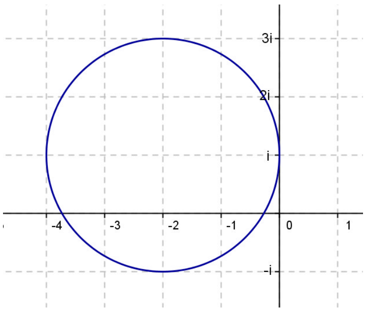
b. $\left| z-2i \right| = 3$ is een cirkel met middelpunt $(0,2i)$ en straal 3



c. $\left| z-1-2i \right| = 3$ is een cirkel met middelpunt $(1,2i)$ en straal 3

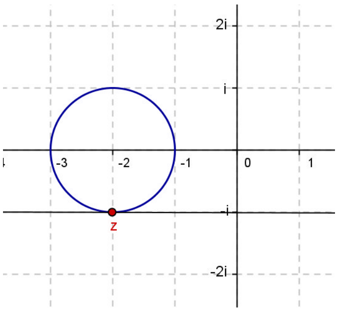


d. $\left| z+2-i \right| = 2$ is een cirkel met middelpunt $(-2,i)$ en straal 2

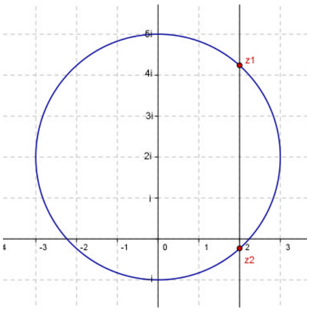


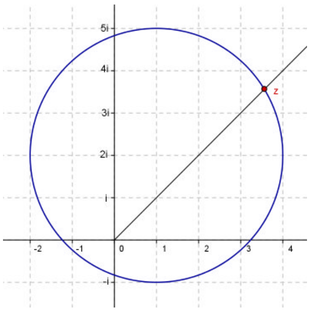
Opgave 27:

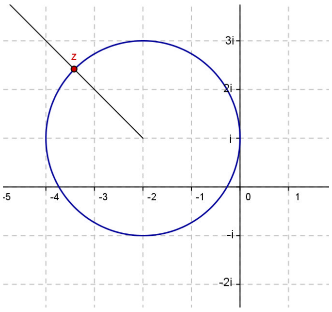
a.



b.



c. 

d. 

Opgave 28:

a. $\tan\varphi_1 = \frac{1}{-1} = -1$ en φ in het 2^e kwadrant, dus $\arg(z_1) = 135°$

$|z_1| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$

b. $\tan\varphi_2 = \dfrac{\sqrt{2}\cdot\sin 135°}{\sqrt{2}\cdot\cos 135°} = \tan 135°$ dus $\arg(z_2) = 135°$

$|z_2| = \sqrt{(\sqrt{2}\cdot COS 135°)^2 + (\sqrt{2}\cdot\sin 135°)^2}$

$= \sqrt{2\cos^2 135° + 2\sin^2 135°}$

$= \sqrt{2(\cos^2 135° + \sin^2 135°)}$

$= \sqrt{2\cdot 1} = \sqrt{2}$

c. $\arg(z_1) = \arg(z_2)$

$|z_1| = |z_2|$

Opgave 29:

a. $|z| = \sqrt{10^2 + 10^2} = \sqrt{200} = 10\sqrt{2}$

$\arg(z) = 45°$

$z = 10\sqrt{2}\cdot(\cos 45° + i\cdot\sin 45°)$

b. $|z| = \sqrt{3^2 + (-4)^2} = 5$

$\arg(z) = -53{,}1°$

$z = 5\cdot(\cos(-53{,}1°) + i\cdot\sin(-53{,}1°))$

c. $|z| = 8$

$\arg(z) = 0°$

$z = 8\cdot(\cos 0° + i\cdot\sin 0°)$

d. $z = \dfrac{1+i}{1-i} = i$

$|z| = 1$
$\arg(z) = 90°$
$z = \cos 90° + i \cdot \sin 90°$

e. $z = (2+i)^2 = 3 + 4i$
$|z| = \sqrt{3^2 + 4^2} = 5$
$\arg(z) = 53{,}1°$
$z = 5 \cdot (\cos 53{,}1° + i \cdot \sin 53{,}1°)$

f. $z = -5i$
$|z| = 5$
$\arg(z) = -90°$
$z = 5 \cdot (\cos(-90°) + i \cdot \sin(-90°))$

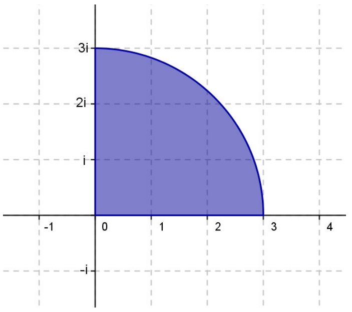
g. $|z| = \sqrt{(-5)^2 + 12^2} = 13$
$\arg(z) = -22{,}6°$
$z = 13 \cdot (\cos(-22{,}6°) + i \cdot \sin(-22{,}6°))$

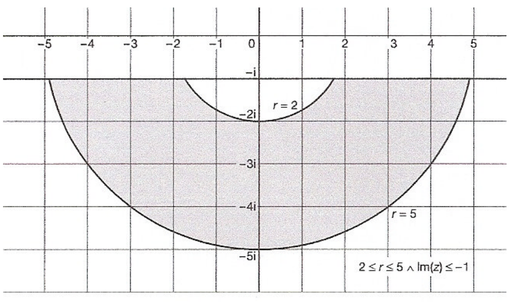
h. $z = \dfrac{12 - 12i}{i} = -12 - 12i$
$|z| = \sqrt{(-12)^2 + (-12)^2} = 12\sqrt{2}$
$\arg(z) = -135°$
$z = 12\sqrt{2} \cdot (\cos(-135°) + i \cdot \sin(-135°))$

Opgave 30:

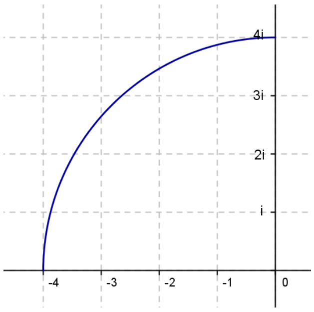
a. $15 \cdot (\cos 30° + i \cdot \sin 30°) = 15 \cdot (\tfrac{1}{2}\sqrt{3} + i \cdot \tfrac{1}{2}) = 7\tfrac{1}{2}\sqrt{3} + 7\tfrac{1}{2}i$
b. $100 \cdot (\cos 90° + i \cdot \sin 90°) = 100 \cdot (0 + i \cdot 1) = 100i$
c. $\sqrt{2} \cdot \cos 135° + i \cdot \sqrt{2} \cdot \sin 135° = \sqrt{2} \cdot -\tfrac{1}{2}\sqrt{2} + i \cdot \sqrt{2} \cdot \tfrac{1}{2}\sqrt{2} = -1 + i$
d. $\sqrt{5} \cdot (\cos(-90°) + i \cdot \sin(-90°)) = \sqrt{5} \cdot (0 + i \cdot -1) = -i\sqrt{5}$

Opgave 31:

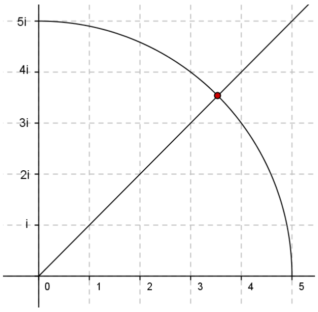
a.


b.


c.



d.



8.3 De formule van De Moivre

Opgave 32:

a. $\cos 12° \cdot \cos 18° - \sin 12° \cdot \sin 18° = 0{,}866$

$\cos 30° = 0{,}866$

b. $\cos 10° \cdot \cos 5° - \sin 10° \cdot \sin 15° = \cos 15°$

Opgave 33:

a. $\sin 12° \cdot \cos 18° + \cos 12° \cdot \sin 18° = 0{,}5$

$\sin 30° = 0{,}5$

b. $\sin 10° \cdot \cos 5° + \cos 10° \cdot \sin 5° = \sin 15°$

Opgave 34:

a. $(\cos 12° + i \cdot \sin 12°)(\cos 18° + i \cdot \sin 18°) = 0{,}866 + 0{,}5i$

$\cos 30° + i \cdot \sin 30° = 0{,}866 + 0{,}5i$

b. $(\cos\alpha + i \cdot \sin\alpha)(\cos\beta + i \cdot \sin\beta) =$

$\cos\alpha \cdot \cos\beta + i^2 \sin\alpha \cdot \sin\beta + \sin\alpha \cdot \cos\beta \cdot i + \cos\alpha \cdot \sin\beta \cdot i =$

$\cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta + i \cdot (\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta) =$

$\cos(\alpha + \beta) + i \cdot \sin(\alpha + \beta)$

Opgave 35:

a. $|(3-3i)(1+i)| = |3-3i| \cdot |1+i| = \sqrt{3^2 + (-3)^2} \cdot \sqrt{1^2 + 1^2} = \sqrt{18} \cdot \sqrt{2} = \sqrt{36} = 6$

$\arg((3-3i)(1+i)) = \arg(3-3i) + \arg(1+i) = -45° + 45° = 0°$

b. $\left|(2-2i) \cdot \sqrt{2} \cdot (\cos 45° + i \cdot \sin 45°)\right| = |2-2i| \cdot \sqrt{2} = \sqrt{2^2 + (-2)^2} \cdot \sqrt{2} = \sqrt{8} \cdot \sqrt{2} = \sqrt{16} = 4$

$\arg((2-2i) \cdot \sqrt{2} \cdot (\cos 45° + i \cdot \sin 45°)) = \arg(2-2i) + \arg(\sqrt{2} \cdot (\cos 45° + i \cdot \sin 45°)) =$

$-45° + 45° = 0°$

c. $\left|\dfrac{8(\cos 12° + i \cdot \sin 12°)}{2(\cos 17° + i \cdot \sin 17°)}\right| = \dfrac{8}{2} = 4$

$\arg\left(\dfrac{8(\cos 12° + i \cdot \sin 12°)}{2(\cos 17° + i \cdot \sin 17°)}\right) = 12° - 17° = -5°$

Opgave 36:

$z = 1 + i$

$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\arg(z) = 45°$

a. $|2z| = 2 \cdot |z| = 2\sqrt{2}$

$\arg(2z) = \arg(2) + \arg(z) = 0° + 45° = 45°$

b. $|i \cdot z| = |i| \cdot |z| = 1 \cdot \sqrt{2} = \sqrt{2}$

$\arg(i \cdot z) = \arg(i) + \arg(z) = 90° + 45° = 135°$

c. $\left|\dfrac{1}{z}\right| = \dfrac{|1|}{|z|} = \dfrac{1}{\sqrt{2}} = \tfrac{1}{2}\sqrt{2}$

$\arg\left(\frac{1}{z}\right) = \arg(1) - \arg(z) = 0° - 45° = -45°$

d. $\left|z^2\right| = \left|z\right|^2 = (\sqrt{2})^2 = 2$

$\arg(z^2) = \arg(z) + \arg(z) = 45° + 45° = 90°$

e. $\left|z^5\right| = \left|z\right|^5 = (\sqrt{2})^5 = 4\sqrt{2}$

$\arg(z^5) = 5 \cdot \arg(z) = 5 \cdot 45° = 225°$

f. $\left|\frac{2i}{z}\right| = \frac{|2i|}{|z|} = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\arg\left(\frac{2i}{z}\right) = \arg(2i) - \arg(z) = 90° - 45° = 45°$

<u>Opgave 37:</u>
I en II

<u>Opgave 38:</u>

a. $z = 1 - i$

$\left|z\right| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\arg(z) = -45°$

$\left|z^6\right| = (\sqrt{2})^6 = 8$

$\arg(z^6) = 6 \cdot -45° = -270° = 90°$

$(1-i)^6 = 8(\cos 90° + i \cdot \sin 90°)$

b. $(\cos 20° + i \cdot \sin 20°)^4 = \cos 80° + i \cdot \sin 80°$

c. $z = 7i$

$\left|z\right| = 7$

$\arg(z) = 90°$

$\left|z^3\right| = 7^3 = 343$

$\arg(z^3) = 3 \cdot 90° = 270°$

$(7i)^3 = 343(\cos 270° + i \cdot \sin 270°)$

<u>Opgave 39:</u>

a. $(\cos 30° + i \cdot \sin 30°)^2 = \cos 60° + i \cdot \sin 60° = \tfrac{1}{2} + \tfrac{1}{2}i\sqrt{3}$

b. $(\cos 45° + i \cdot \sin 45°)^5 = \cos 225° + i \cdot \sin 225° = -\tfrac{1}{2}\sqrt{2} - \tfrac{1}{2}i\sqrt{2}$

c. $(\cos 300° + i \cdot \sin 300°)^{-5} = \cos(-60°) + i \cdot \sin(-60°) = \tfrac{1}{2} - \tfrac{1}{2}i\sqrt{3}$

d. $(\tfrac{1}{2}\sqrt{2} - \tfrac{1}{2}i\sqrt{2})^5 = (\cos(-45°) + i \cdot \sin(-45°))^5 = \cos(-225°) + i \cdot \sin(-225°) =$

$-\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}i\sqrt{2}$

e. $(\tfrac{1}{2}\sqrt{3} + \tfrac{1}{2}i)^{10} = (\cos 30° + i \cdot \sin 30°)^{10} = \cos 300° + i \cdot \sin 300° = \tfrac{1}{2} - \tfrac{1}{2}i\sqrt{3}$

Opgave 40:

a. $(\cos x + i \cdot \sin x)^3 =$

$\binom{3}{0} \cdot \cos^3 x + \binom{3}{1} \cdot \cos^2 x \cdot i \cdot \sin x + \binom{3}{2} \cdot \cos x \cdot i^2 \cdot \sin^2 x + \binom{3}{3} \cdot i^3 \cdot \sin^3 x =$

$\cos^3 x + 3i \cdot \cos^2 x \cdot \sin x - 3\cos x \cdot \sin^2 x - i \cdot \sin^3 x$

b. als je in de formule van De Moivre $n = 3$ neemt en $\varphi = x$ krijg je

$(\cos x + i \cdot \sin x)^3 = \cos 3x + i \cdot \sin 3x$

c. $\cos 3x = \cos^3 x - 3\cos x \cdot \sin^2 x$

$\sin 3x = 3\cos^2 x \cdot \sin x - \sin^3 x$

d. $(\cos x + i \cdot \sin x)^2 = \cos^2 x + 2i \cdot \cos x \cdot \sin x - \sin^2 x$

$(\cos x + i \cdot \sin x)^2 = \cos 2x + i \cdot \sin 2x$

$\cos 2x = \cos^2 x - \sin^2 x$

$\sin 2x = 2\cos x \cdot \sin x$

e. $(\cos x + i \cdot \sin x)^4 =$

$\cos^4 x + 4i \cdot \cos^3 x \cdot \sin x - 6\cos^2 x \cdot \sin^2 x - 4i \cdot \cos x \cdot \sin^3 x + \sin^4 x$

$(\cos x + i \cdot \sin x)^4 = \cos 4x + i \cdot \sin 4x$

$\cos 4x = \cos^4 x - 6\cos^2 x \cdot \sin^2 x + \sin^4 x$

$\sin 4x = 4\cos^3 x \cdot \sin x - 4\cos x \cdot \sin^3 x$

Opgave 41:

a. $4i = 4 \cdot (0 + i \cdot 1) = 4(\cos 90° + i \cdot \sin 90°) = 4(\cos(-270°) + i \cdot \sin(-270°))$

b. $z^2 = 4i$

$z = (4i)^{\frac{1}{2}} = (4(\cos 90° + i \cdot \sin 90°))^{\frac{1}{2}} = 2(\cos 45° + i \cdot \sin 45°) = 2(\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}) =$

$\sqrt{2} + i\sqrt{2}$

of $z = (4i)^{\frac{1}{2}} = (4(\cos(-270°) + i \cdot \sin(-270°)))^{\frac{1}{2}} = 2(\cos(-135°) + i \cdot \sin(-135°)) =$

$2(-\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}) = -\sqrt{2} - i\sqrt{2}$

c. $\cos 450° = \cos 90°$ en $\sin 450° = \sin 90°$

dus dit levert weer $\sqrt{2} + i\sqrt{2}$ op

Opgave 42:

a. $|z^3| = |z|^3 = 1^3 = 1$

$\arg(z^3) = 3 \cdot \arg(z) = 3 \cdot 120° = 360°$

dus $z^3 = 1 \cdot (\cos 360° + i \cdot \sin 360°) = 1 \cdot (1 + i \cdot 0) = 1$

b. $240°$ en $360°$

Opgave 43:

a. $z^2 = -4i$

$|z^2| = 4$ dus $|z| = 2$

$\arg(z^2) = -90° + k \cdot 360°$

$\arg(z) = -45° + k \cdot 180°$

$z = 2(\cos(-45°) + i \cdot \sin(-45°)) = 2 \cdot (\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}) = \sqrt{2} - i\sqrt{2}$

of $z = 2(\cos 135° + i \cdot \sin 135°) = 2 \cdot (-\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}i\sqrt{2}) = -\sqrt{2} + i\sqrt{2}$

b. $z^2 = 9i$

$\left|z^2\right| = 9$ dus $\left|z\right| = 3$

$\arg(z^2) = 90° + k \cdot 360°$

$\arg(z) = 45° + k \cdot 180°$

$z = 3(\cos 45° + i \cdot \sin 45°) = 3(\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}i\sqrt{2}) = 1\tfrac{1}{2}\sqrt{2} + 1\tfrac{1}{2}i\sqrt{2}$

of $z = 3(\cos 225° + i \cdot \sin 225°) = 3(-\tfrac{1}{2}\sqrt{2} - \tfrac{1}{2}i\sqrt{2}) = -1\tfrac{1}{2}\sqrt{2} - 1\tfrac{1}{2}i\sqrt{2}$

c. $z^3 = 27$

$\left|z^3\right| = 27$ dus $\left|z\right| = \sqrt[3]{27} = 3$

$\arg(z^3) = 0° + k \cdot 360°$

$\arg(z) = 0° + k \cdot 120°$

$z = 3(\cos 0° + i \cdot \sin 0°) = 3(1 + i \cdot 0) = 3$

of $z = 3(\cos 120° + i \cdot \sin 120°) = 3(-\tfrac{1}{2} + \tfrac{1}{2}i\sqrt{3}) = -1\tfrac{1}{2} + 1\tfrac{1}{2}i\sqrt{3}$

of $z = 3(\cos 240° + i \cdot \sin 240°) = 3(-\tfrac{1}{2} - \tfrac{1}{2}i\sqrt{3}) = -1\tfrac{1}{2} - 1\tfrac{1}{2}i\sqrt{3}$

d. $z^4 = -81$

$\left|z^4\right| = 81$ dus $\left|z\right| = \sqrt[4]{81} = 3$

$\arg(z^4) = 180° + k \cdot 360°$

$\arg(z) = 45° + k \cdot 90°$

$z = 3(\cos 45° + i \cdot \sin 45°) = 3(\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}i\sqrt{2}) = 1\tfrac{1}{2}\sqrt{2} + 1\tfrac{1}{2}i\sqrt{2}$

of $z = 3(\cos 135° + i \cdot \sin 135°) = 3(-\tfrac{1}{2}\sqrt{2} + \tfrac{1}{2}i\sqrt{2}) = -1\tfrac{1}{2}\sqrt{2} + 1\tfrac{1}{2}i\sqrt{2}$

of $z = 3(\cos 225° + i \cdot \sin 225°) = 3(-\tfrac{1}{2}\sqrt{2} - \tfrac{1}{2}i\sqrt{2}) = -1\tfrac{1}{2}\sqrt{2} - 1\tfrac{1}{2}i\sqrt{2}$

of $z = 3(\cos 315° + i \cdot \sin 315°) = 3(\tfrac{1}{2}\sqrt{2} - \tfrac{1}{2}i\sqrt{2}) = 1\tfrac{1}{2}\sqrt{2} - 1\tfrac{1}{2}i\sqrt{2}$

Opgave 44:

a. $z^2 = 2 + i$

$\left|z^2\right| = \sqrt{2^2 + 1^2} = \sqrt{5}$

$\left|z\right| = \sqrt{\sqrt{5}} = 1{,}495$

$\arg(z^2) = 26{,}6° + k \cdot 360°$

$\arg(z) = 13{,}3° + k \cdot 180°$

$z = 1{,}495(\cos 13{,}3° + i \cdot \sin 13{,}3°) = 1{,}46 + 0{,}34i$

of $z = 1{,}495(\cos 193{,}3° + i \cdot \sin 193{,}3°) = -1{,}46 - 0{,}34i$

b. $z^2 = -4 + 3i$

$\left|z^2\right| = \sqrt{(-4)^2 + 3^2} = 5$

$\left|z\right| = \sqrt{5}$

$\arg(z^2) = 143{,}1° + k \cdot 360°$

$\arg(z) = 71{,}6° + k \cdot 180°$

$z = \sqrt{5} \cdot (\cos 71{,}6° + i \cdot \sin 71{,}6°) = 0{,}71 + 2{,}12i$

of $z = \sqrt{5} \cdot (\cos 251{,}6° + i \cdot \sin 251{,}6°) = -0{,}71 - 2{,}12\,i$

c. $z^3 = -6 + 3i$

$|z^3| = \sqrt{(-6)^2 + 3^2} = \sqrt{45}$

$|z| = \sqrt[3]{\sqrt{45}} = 1{,}886$

$\arg(z^3) = 153{,}4° + k \cdot 360°$

$\arg(z) = 51{,}1° + k \cdot 120°$

$z = 1{,}886(\cos 51{,}1° + i \cdot \sin 51{,}1°) = 1{,}18 + 1{,}47\,i$

of $z = 1{,}886(\cos 171{,}1° + i \cdot \sin 171{,}1°) = -1{,}86 + 0{,}29\,i$

of $z = 1{,}886(\cos 291{,}1° + i \cdot \sin 291{,}1°) = 0{,}68 - 1{,}76\,i$

d. $z^4 = 10$

$|z^4| = 10$

$|z| = \sqrt[4]{10}$

$\arg(z^4) = 0° + k \cdot 360°$

$\arg(z) = 0° + k \cdot 90°$

$z = \sqrt[4]{10} \cdot (\cos 0° + i \cdot \sin 0°) = 1{,}78$

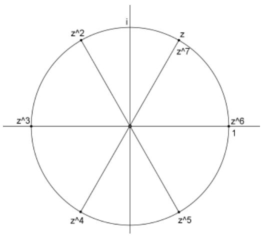
of $z = \sqrt[4]{10} \cdot (\cos 90° + i \cdot \sin 90°) = 1{,}78\,i$

of $z = \sqrt[4]{10} \cdot (\cos 180° + i \cdot \sin 180°) = -1{,}78$

of $z = \sqrt[4]{10} \cdot (\cos 270° + i \cdot \sin 270°) = -1{,}78\,i$

Opgave 45:

a.



b. $|z| = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2}\sqrt{3})^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$

$\arg(z) = 60°$

$\arg(z^n) = n \cdot 60°$

$M(0,0)$ en $r = 1$

c. $z = \frac{1}{2} + \frac{1}{2}i\sqrt{3} \quad \vee \quad z = -\frac{1}{2} - \frac{1}{2}i\sqrt{3}$

d. $z = \frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2} \quad \vee \quad z = -\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2} \quad \vee \quad z = -\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2} \quad \vee \quad z = \frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}$

Opgave 46:

a. $(z-1)^2 = \frac{1}{2} + \frac{1}{2}i\sqrt{3}$

$u^2 = \frac{1}{2} + \frac{1}{2}i\sqrt{3}$

$|u^2| = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2}\sqrt{3})^2} = 1$

$|u| = 1$

$\arg(u^2) = 60° + k \cdot 360°$
$\arg(u) = 30° + k \cdot 180°$
$u = 1 \cdot (\cos 30° + i \cdot \sin 30°) = \frac{1}{2}\sqrt{3} + \frac{1}{2}i \quad \vee \quad u = 1 \cdot (\cos 210° + i \cdot \sin 210°) = -\frac{1}{2}\sqrt{3} - \frac{1}{2}i$
$z - 1 = \frac{1}{2}\sqrt{3} + \frac{1}{2}i \quad \vee \quad z - 1 = -\frac{1}{2}\sqrt{3} - \frac{1}{2}i$
$z = 1 + \frac{1}{2}\sqrt{3} + \frac{1}{2}i \quad \vee \quad z = 1 - \frac{1}{2}\sqrt{3} - \frac{1}{2}i$

b. $(z - 1 - i)^2 = -i$
$u^2 = -i$
$|u^2| = 1$
$|u| = 1$
$\arg(u^2) = 270° + k \cdot 360°$
$\arg(u) = 135° + k \cdot 180°$
$u = \cos 135° + i \cdot \sin 135° = -\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2} \quad \vee \quad u = \cos 315° + i \cdot \sin 315° = \frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}$
$z - 1 - i = -\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2} \quad \vee \quad z - 1 - i = \frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}$
$z = 1 - \frac{1}{2}\sqrt{2} + i \cdot (1 + \frac{1}{2}\sqrt{2}) \quad \vee \quad z = 1 + \frac{1}{2}\sqrt{2} + i \cdot (1 - \frac{1}{2}\sqrt{2})$

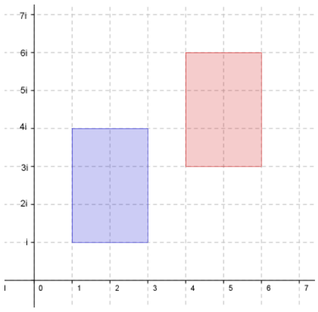
c. $z^2 - 4z + 4 = \frac{1}{2} - \frac{1}{2}i\sqrt{3}$
$(z - 2)^2 = \frac{1}{2} - \frac{1}{2}i\sqrt{3}$
$u^2 = \frac{1}{2} - \frac{1}{2}i\sqrt{3}$
$|u^2| = \sqrt{(\frac{1}{2})^2 + (-\frac{1}{2}\sqrt{3})^2} = 1$
$|u| = 1$
$\arg(u^2) = 300° + k \cdot 360°$
$\arg(u) = 150° + k \cdot 180°$
$u = \cos 150° + i \cdot \sin 150° = -\frac{1}{2}\sqrt{3} + \frac{1}{2}i \quad \vee \quad u = \cos 330° + i \cdot \sin 330° = \frac{1}{2}\sqrt{3} - \frac{1}{2}i$
$z - 2 = -\frac{1}{2}\sqrt{3} + \frac{1}{2}i \quad \vee \quad z - 2 = \frac{1}{2}\sqrt{3} - \frac{1}{2}i$
$z = 2 - \frac{1}{2}\sqrt{3} + \frac{1}{2}i \quad \vee \quad z = 2 + \frac{1}{2}\sqrt{3} - \frac{1}{2}i$

d. $z^2 - 6z + 10 = i\sqrt{3}$
$(z - 3)^2 + 1 = i\sqrt{3}$
$(z - 3)^2 = -1 + i\sqrt{3}$
$u^2 = -1 + i\sqrt{3}$
$|u^2| = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$
$|u| = \sqrt{2}$
$\arg(u^2) = 120° + k \cdot 360°$
$\arg(u) = 60° + k \cdot 180°$
$u = \sqrt{2} \cdot (\cos 60° + i \cdot \sin 60°) = \sqrt{2} \cdot (\frac{1}{2} + \frac{1}{2}i\sqrt{3}) = \frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{6}$
of $u = \sqrt{2} \cdot (\cos 240° + i \cdot \sin 240°) = \sqrt{2} \cdot (-\frac{1}{2} - \frac{1}{2}i\sqrt{3}) = -\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{6}$
$z - 3 = \frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{6} \quad \vee \quad z - 3 = -\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{6}$
$z = 3 + \frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{6} \quad \vee \quad z = 3 - \frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{6}$

8.4 Complexe functies

Opgave 47:

a.



b. $z=1+i$ geeft $4+3i$

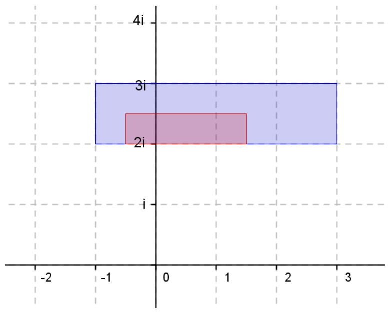
$z=3+i$ geeft $6+3i$

$z=3+4i$ geeft $6+6i$

$z=1+4i$ geeft $4+6i$

c. de rechthoek is getransleerd over $(3,2)$

Opgave 48:



a. $f(-1+2i)=-\frac{1}{2}+2i$

$f(3+2i)=1\frac{1}{2}+2i$

$f(3+3i)=1\frac{1}{2}+2\frac{1}{2}i$

$f(-1+3i)=-\frac{1}{2}+2\frac{1}{2}i$

Dus de rechthoek wordt eerst vermenigvuldigt t.o.v. O met de factor $\frac{1}{2}$ en daarna getransleerd over $(0,1)$

b. $f(x)=\frac{1}{2}z+i=0$

$\frac{1}{2}z=-i$

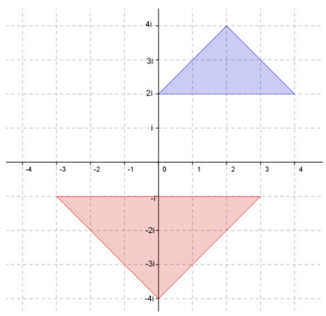
$z=-2i$

c. $f(x)=\frac{1}{2}z+i=z$

$-\frac{1}{2}z=-i$

$z=2i$

Opgave 49:



a. $f(2i)=3-i$

$f(4+2i)=-3-i$

$f(2+4i)=-4i$

1. vermenigvuldiging t.o.v. O met factor $-1\frac{1}{2}$
2. translatie over $(3,2)$

b. $f(z)=-1\frac{1}{2}z+3+2i=0$

$-1\frac{1}{2}z=-3-2i$

$z=2+\frac{4}{3}i$

c. $f(z)=-1\frac{1}{2}z+3+2i=z$

$-2\frac{1}{2}z=-3-2i$

$z=\frac{6}{5}+\frac{4}{5}i$

Opgave 50:

a. nulpunt: $f(z) = 3z + 2 - 4i = 0$

$3z = -2 + 4i$

$z = -\frac{2}{3} + \frac{4}{3}i$

dekpunt: $f(z) = 3z + 2 - 4i = z$

$2z = -2 + 4i$

$z = -1 + 2i$

b. nulpunt: $g(z) = \frac{1}{3}z + 5 = 0$

$\frac{1}{3}z = -5$

$z = -15$

dekpunt: $g(z) = \frac{1}{3}z + 5 = z$

$-\frac{2}{3}z = -5$

$z = 7\frac{1}{2}$

Opgave 51:

a. $f(z) = az + 5 - 2i = z$

$az - z = -5 + 2i$

$(a-1)z = -5 + 2i$

$z = \dfrac{-5 + 2i}{a - 1}$ als $a \neq 1$

dus er is geen dekpunt als $a = 1$

b. $f(z) = az + 5 - 2i = 0$

$az = -5 + 2i$

$z = \dfrac{-5 + 2i}{a}$ als $a \neq 0$

dus er is geen nulpunt als $a = 0$

Opgave 52:

a. $f(1+2i) = 3(1+2i) + a + bi = 0$

$3 + 6i + a + bi = 0$

$3 + a + (6+b)i = 0$

$$\begin{cases} 3 + a = 0 \\ 6 + b = 0 \end{cases}$$

$a = -3 \quad \wedge \quad b = -6$

b. $f(1+2i) = 3(1+2i) + a + bi = 1 + 2i$

$3 + 6i + a + bi = 1 + 2i$

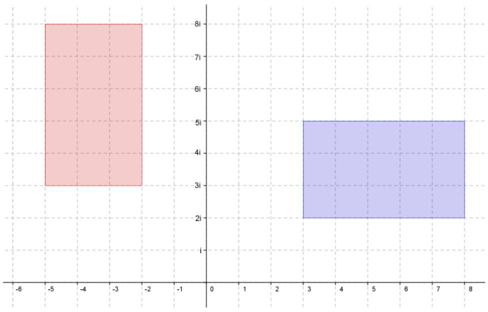
$3 + a + (6+b)i = 1 + 2i$

$$\begin{cases} 3 + a = 1 \\ 6 + b = 2 \end{cases}$$

$a = -2 \quad \wedge \quad b = -4$

Opgave 53:

a.



b. $z = 3 + 2i$ geeft $-2 + 3i$

$z = 8 + 2i$ geeft $-2 + 8i$

$z = 8 + 5i$ geeft $-5 + 8i$

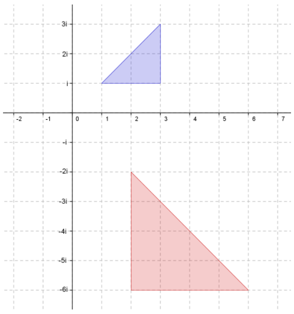
$z = 3 + 5i$ geeft $-5 + 3i$

c. rotatie om O over $90°$

d. $\arg(\frac{1}{2} + \frac{1}{2}i\sqrt{3}) = 60°$ dus rotatie om O over $60°$

Opgave 54:

a.



b. $z = 1 + i$ geeft $2 - 2i$

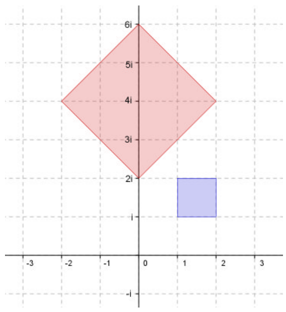
$z = 3 + i$ geeft $2 - 6i$

$z = 3 + 3i$ geeft $6 - 6i$

c. rotatie om O over $-90°$

vermenigvuldiging t.o.v. O met factor 2

Opgave 55:



a. $|2 + 2i| = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

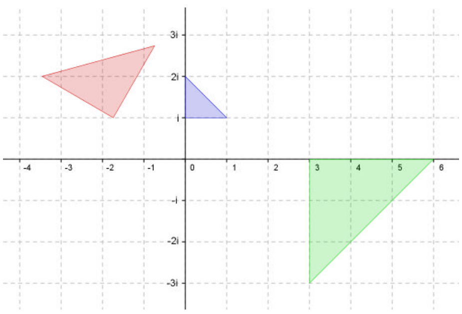
$\arg(2 + 2i) = 45°$

rotatie om O over $45°$

vermenigvuldiging t.o.v. O met $2\sqrt{2}$

b. het bereik is $2\sqrt{2} \le |z| \le 4\sqrt{2} \quad \wedge \quad 75° \le \arg(z) \le 105°$

Opgave 56:



a. $f(i) = -\sqrt{3} + i$

$f(1 + i) = 1 - \sqrt{3} + i \cdot (1 + \sqrt{3})$

$f(2i) = -2\sqrt{3} + 2i$

$\arg(1 + i\sqrt{3}) = 60°$

$\left|1+i\sqrt{3}\right| = \sqrt{1^2+(\sqrt{3})^2} = 2$

rotatie om O over $60°$

vermenigvuldiging t.o.v. O met factor 2

b. $g(i)=3$

$g(1+i)=3-3i$

$g(2i)=6$

rotatie om O over $-90°$

vermenigvuldiging t.o.v. O met factor 3

Opgave 57:

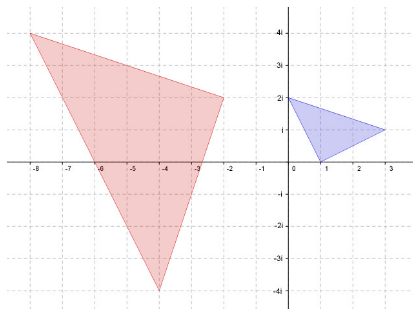
$\arg(\sqrt{3}-i) = -30°$

$\left|\sqrt{3}-i\right| = \sqrt{(\sqrt{3})^2+(-1)^2} = 2$

a. het bereik is $20 \le |z| \le 40 \quad \wedge \quad 60° \le \arg(z) \le 150°$

b. het bereik is $|z| \ge 6 \quad \wedge \quad -30° \le \arg(z) \le 60°$

Opgave 58:



a. $f(1) = -2+2i$

$f(3+i) = -8+4i$

$f(2i) = -4-4i$

$\arg(-2+2i) = 135°$

$\left|-2+2i\right| = \sqrt{(-2)^2+2^2} = 2\sqrt{2}$

rotatie om O over $135°$

vermenigvuldiging t.o.v. O met factor $2\sqrt{2}$

b. $(-2+2i)\cdot z = 3+2i$

$$z = \frac{3+2i}{-2+2i} = \frac{3+2i}{-2+2i}\cdot\frac{-2-2i}{-2-2i} = \frac{-2-10i}{8} = -\tfrac{1}{4} - 1\tfrac{1}{4}i$$

c. $(-2+2i)\cdot z = \dfrac{-2+2i}{z}$

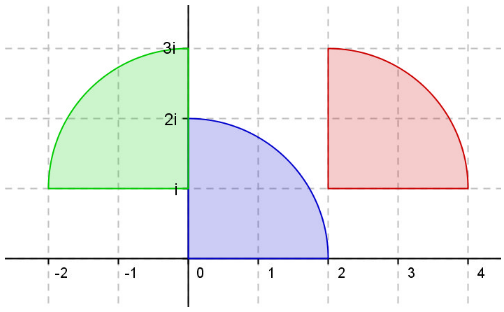
$z = \dfrac{1}{z}$

$z^2 = 1$

$z = 1 \quad \vee \quad z = -1$

Opgave 59:

a.



b. translatie over $(2,1)$

c. rotatie om O over $90°$

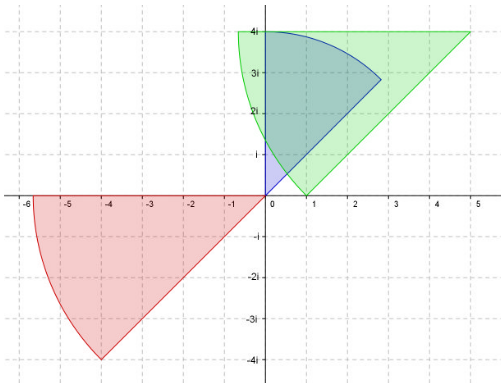
translatie over $(0,1)$

Opgave 60:

a. $(-1+i)\cdot z+5+4i=10+i$

$(-1+i)\cdot z=5-3i$

$z=\dfrac{5-3i}{-1+i}=\dfrac{5-3i}{-1+i}\cdot\dfrac{-1-i}{-1-i}=\dfrac{-8-2i}{2}=-4-i$



b. $\arg(-1+i)=135°$

$|-1+i|=\sqrt{(-1)^2+1^2}=\sqrt{2}$

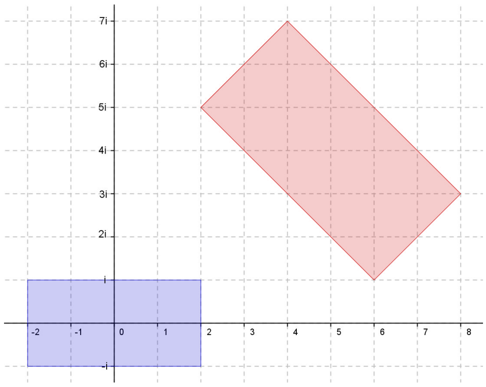
1. rotatie om O over $135°$
2. vermenigvuldiging t.o.v. O met factor $\sqrt{2}$
3. translatie over $(5,4)$

c. $f(2+i)=2+5i$

$f(-2+i)=6+i$

$f(-2-i)=8+3i$

$f(2-i)=4+7i$



d. de vermenigvuldigingsfactor is $\sqrt{2}$

dus de oppervlakte wordt $(\sqrt{2})^2=2\times$ zo groot

$Opp(V)=\dfrac{10}{2}=5$

Opgave 61:

a. nulpunt: $f(z)=(1+i\sqrt{3})\cdot z-2+i=0$

$(1+i\sqrt{3})\cdot z=2-i$

$z=\dfrac{2-i}{1+i\sqrt{3}}=\dfrac{2-i}{1+i\sqrt{3}}\cdot\dfrac{1-i\sqrt{3}}{1-i\sqrt{3}}=\dfrac{2-\sqrt{3}+i(-1-2\sqrt{3})}{4}=\frac{1}{2}-\frac{1}{4}\sqrt{3}+(-\frac{1}{4}-\frac{1}{2}\sqrt{3})\cdot i$

dekpunt: $f(z)=(1+i\sqrt{3})\cdot z-2+i=z$

$(1+i\sqrt{3})\cdot z-z=2-i$

$i\sqrt{3}\cdot z=2-i$

$z=\dfrac{2-i}{i\sqrt{3}}=\dfrac{2-i}{i\sqrt{3}}\cdot\dfrac{i\sqrt{3}}{i\sqrt{3}}=\dfrac{2i\sqrt{3}+\sqrt{3}}{-3}=-\frac{1}{3}\sqrt{3}-\frac{2}{3}i\sqrt{3}$

b. nulpunt: $g(z)=-2i\cdot z+1-3i=0$

$-2i\cdot z=-1+3i$

$z=\dfrac{-1+3i}{-2i}=\dfrac{-1+3i}{-2i}\cdot\dfrac{i}{i}=\dfrac{-i-3}{2}=-1\frac{1}{2}-\frac{1}{2}i$

dekpunt: $g(z)=-2i\cdot z+1-3i=z$

$-2i\cdot z-z=-1+3i$

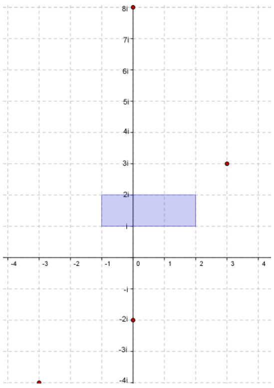
$2i\cdot z+z=1-3i$

$(1+2i)\cdot z=1-3i$

$z=\dfrac{1-3i}{1+2i}=\dfrac{1-3i}{1+2i}\cdot\dfrac{1-2i}{1-2i}=\dfrac{-5-5i}{5}=-1-i$

Opgave 62:

a.



b. $f(-1+i)=(-1+i)^2=1-2i+i^2=-2i$

$f(2+i)=(2+i)^2=4+4i+i^2=3+4i$

$f(2+2i)=(2+2i)^2=4+8i+4i^2=8i$

$f(-1+2i)=(-1+2i)^2=1-4i+4i^2=-3-4i$

c. nee

Opgave 63:

a. $f(z)=z^2+2=0$

$z^2=-2$

$z=\sqrt{-2}=i\sqrt{2} \quad \vee \quad z=-i\sqrt{2}$

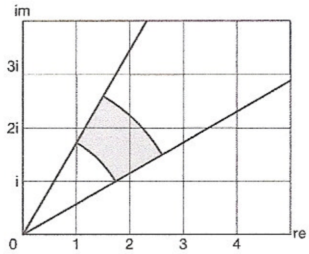
b. $f(z)=z^2+2=z$

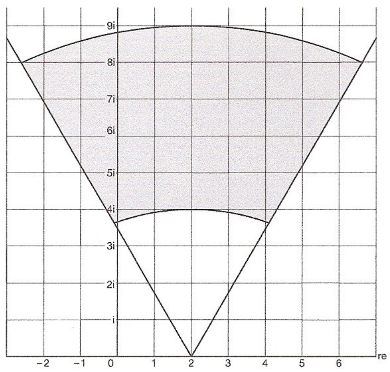
$z^2-z+2=0$

$z=\dfrac{1\pm\sqrt{-7}}{2}=\tfrac{1}{2}\pm\tfrac{1}{2}i\sqrt{7}$

$z=\tfrac{1}{2}+\tfrac{1}{2}i\sqrt{7} \quad \vee \quad z=\tfrac{1}{2}-\tfrac{1}{2}i\sqrt{7}$

c.





d. $f(1-3i)=-6-6i$

$f(1-2i)=-1-4i$

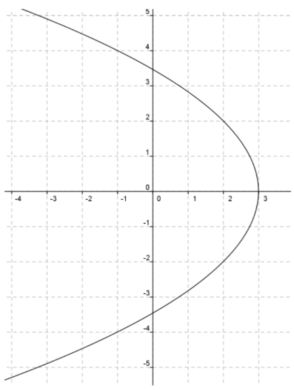
$f(1-i)=2-2i$

$f(1)=3$

$f(1+i)=2+2i$

$f(1+2i)=-1+4i$

$f(1+3i)=-6+6i$



Opgave 64:

a. $f(z) = i \cdot z^2 - 4 = 0$

$i \cdot z^2 = 4$

$z^2 = \frac{4}{i} = \frac{4}{i} \cdot \frac{i}{i} = \frac{4i}{-1} = -4i$

$|z^2| = 4$

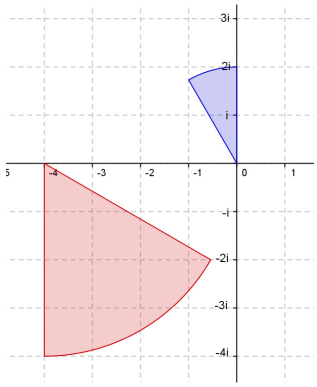
$|z| = 2$

$\arg(z^2) 270° + k \cdot 360°$

$\arg(z) = 135° + k \cdot 180°$

$z = 2(\cos 135° + i \cdot \sin 135°) = 2(-\frac{1}{2}\sqrt{2} + \frac{1}{2}i\sqrt{2}) = -\sqrt{2} + i\sqrt{2}$

of $z = 2(\cos 315° + i \cdot \sin 315°) = 2(\frac{1}{2}\sqrt{2} - \frac{1}{2}i\sqrt{2}) = \sqrt{2} - i\sqrt{2}$



b. 1. $|z|$ wordt gekwadrateerd
   $\arg(z)$ wordt verdubbeld
2. rotatie om O over $90°$
3. translatie over $(-4,0)$

c. $f(1-i) = i \cdot (1-i)^2 - 4 = i \cdot (1 - 2i - 1) - 4 = -2i^2 - 4 = -2$

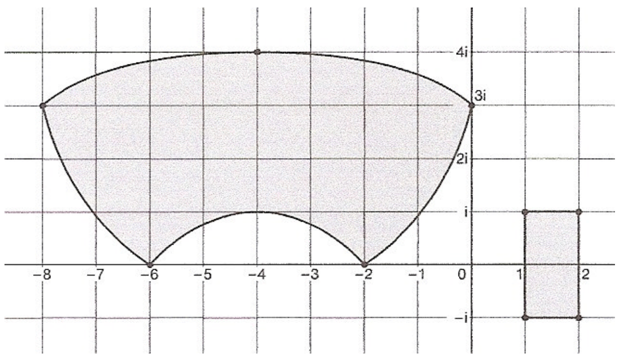
$f(2-i) = i \cdot (2-i)^2 - 4 = i \cdot (4 - 4i - 1) - 4 = 3i - 4i^2 - 4 = 3i$

$f(2+i) = i \cdot (2+i)^2 - 4 = i \cdot (4 + 4i - 1) - 4 = 3i + 4i^2 - 4 = -8 + 3i$

$f(1+i) = i \cdot (1+i)^2 - 4 = i \cdot (1 + 2i - 1) - 4 = 2i^2 - 4 = -6$

$f(2) = i \cdot 2^2 - 4 = -4 + 4i$

$f(1) = i \cdot 1^2 - 4 = -4 + i$



Opgave 65:

a. het beeld van $\text{Re}(z) = 1$ en $\text{Re}(z) = -\text{Im}(z)$ gaat door $-2-2i$

dus $z = 1 - i$

b. $|z^3| = |z|^3$

$\arg(z^3) = 3 \cdot \arg(z)$

Voor ieder punt op een lijn door O wordt het argument $3\times$ zo groot, dus het beeld van een lijn is dus weer een rechte lijn.

c. voor deze punten moet gelden: $\arg(z) = 60° \quad \vee \quad \arg(z) = -60°$

dus $z = 1 + i\sqrt{3} \quad \vee \quad z = 1 - i\sqrt{3}$